An Optimized Intruder Model for SAT-based Model-Checking of Security Protocols

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ARSPA Workshop - IJCAR, Cork, 04 Jul 2004

AVISPA
Automated Validation of Internet Security
Protocols and Applications (IST-2001-39252)

The EU Calculemus
Training Network
(HPRN-CT-2000-00102)
Motivations

- **Context:** Dramatic speed-up of SAT solvers in the last decade: problems with thousands of variables are now solved routinely in milliseconds.

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We proposed reductions of protocol (in)security problems to SAT that can be used to effectively find attacks on small and medium size protocols.

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- **Optimization:** In this work we propose an optimized intruder model that leads in many cases to shorter attacks which can be detected in our framework by generating smaller propositional formulae.
Roadmap

• Protocol Analysis

• Modeling via a simple example:
  – Standard Model
  – Axioms and the Optimized Intruder

• Protocol Insecurity Problems with Axioms

• Encoding Protocol Insecurity Problems with Axioms into SAT

• Implementations and Results

• Conclusions and Perspectives
Protocol Analysis: Modeling

- Protocol as a state transition system in which states correspond to information possessed by participating agents.

- Perfect cryptography: an encrypted message can be neither altered nor read without the appropriate key.

- The Dolev-Yao intruder:
  - controls all the traffic in the network;
  - can compose and send fraudulent messages from the knowledge he can glean from the observed traffic and his own initial knowledge.
Protocol Analysis: Security Problems

• Specified by means of the IF rule-based language suitable for security protocols:
  – **state**: set of facts;
  – **transition relation**: labeled rewrite rules.

• Security requirements such as **authentication** and **secrecy** are reduced to **reachability problems** on this model.

• We focus on **reachability problem with finite number of sessions**.

• This is adequate in practice as attacks on well-known protocols often exploit a small number of sessions.
Modeling: Needham-Schroeder authentication prot. (1)

Let us consider the well known NSPK protocol:

1. $A \rightarrow B : \{A, N_A\}_{K_B}$
2. $B \rightarrow A : \{N_A, N_B\}_{K_A}$
3. $A \rightarrow B : \{N_B\}_{K_B}$

**Scenario:** two concurrent sessions of the protocol

**session 1:** $a$ talks to the intruder $i$;

**session 2:** $a$ talks to $b$.

**Security Requirement:** $B$ authenticates $A$ on $N_A$. 
Modeling: Needham-Schroeder authentication prot. (2)

States are represented as sets of the following facts:

- $fresh(N)$ means that the nonce $N$ has not been used yet.
- $ik(T)$ means that the intruder knows $T$.
- $m(J, S, R, T)$ means that sender $S$ has (supposedly) sent message $T$ to principal $R$ at protocol step $J$.
- $w(J, S, R, [T_1, \ldots, T_k], C)$ represents the state of principal $R$ at step $J$ of session $C$; it means that $R$
  - knows the terms stored in the lists $[T_1, \ldots, T_k]$, and
  - is waiting for a message from $S$ (if $J \neq 0$).
Modeling: Needham-Schroeder authentication prot. (3)

• Initial State:

\[ w(0, a, a, [a, i, ka, ka^{-1}, ki], 1) \]
\[ w(0, a, a, [a, b, ka, ka^{-1}, kb], 2) \]
\[ fresh(nc(n1, 1)) \]
\[ fresh(nc(n1, 2)) \]
\[ fresh(nc(n2, 2)) \]
\[ ik(i) \cdot ik(a) \cdot ik(b) \cdot ik(ki) \cdot ik(ki^{-1}) \cdot ik(ka) \cdot ik(kb) \]

• Bad States:

\[ w(0, a, a, [], [a, b, ka, kb, ka^{-1}], s(1)) \]
\[ w(1, a, b, [], [b, a, kb, ka, kb^{-1}], 1) \]
Modeling: Needham-Schroeder authentication prot. (4)

- Labeled Rewrite Rules:
  - Behaviour of Honest Participants:
    
    \[
    \text{fresh}(\text{nc}(n1, S)). \\
    w(0, A, A, [A, B, Ka, Ka^{-1}, Kb], S) \xrightarrow{\text{step}_0(A, B, Ka, Kb, S)} \\
    w(2, B, A, [\text{nc}(n1, S), A, B, Ka, Ka^{-1}, Kb], S). \\
    m(1, A, B, \{A, \text{nc}(n1, S)\} \times Kb)
    \]
  - Behaviour of the Intruder:
    
    \[
    m(J, S, R, M) \xrightarrow{\text{divert}(J, M, R, S)} ik(S) \\
    ik(\{M\}_K) \cdot ik(K^{-1}) \xrightarrow{\text{decrypt}(K, M)} ik(M) \cdot ik(\{M\}_K) \cdot ik(K^{-1})
    \]
Modeling: Needham-Schroeder authentication prot. (5)

The attack on the simple NSPK protocol

\[(1.1) \quad a \rightarrow i : \{a, na\}_{ki} \quad \text{snd}\]
\[(2.1) \quad i(a) \rightarrow b : \{a, na\}_{kb} \quad rc + dec + dec + snd\]
\[(2.2) \quad b \rightarrow i(a) : \{na, nb\}_{ka} \quad rc_{-snd}\]
\[(1.2) \quad i \rightarrow a : \{na, nb\}_{ka}\]
\[(1.3) \quad a \rightarrow i : \{nb\}_{ki} \quad rc_{-snd}\]
\[(2.3) \quad i(a) \rightarrow b : \{nb\}_{kb} \quad rc + dec + snd\]

requires 3 intruder knowledge manipulations (\textit{dec}) to be executed.

For industrial-scale security protocols in which messages can have a complex structure, such a number can be much more significant.

\textbf{Question:} can we save such decomposing transitions?
Modeling: Axioms and the Optimized Intruder

**Axiom:** formula that states a relation between facts of the transition system and that holds at each state of the transition system.

Axioms are particularly suited to represent relations between intruder knowledge facts. E.g.

\[ ik(\{ M \}_K) \land ik(K^{-1}) \supset ik(M) \]

“Every time the intruder knows \( \{ M \}_K \) and \( K^{-1} \), then it knows instantaneously also \( M \).”

**Idea:** optimize the intruder by replacing decomposing rules with appropriate decomposing axioms.
Protocol Insecurity Problems with Axioms (1)

A Protocol Insecurity Problem (PIP) with axioms is a tuple $\Xi = \langle F, L, R, A, I, G \rangle$ where:

- $F$ and $L$ are sets of atomic formula of sorted 1\textsuperscript{st}-order languages called facts and rule labels, respectively;
- $R$ is a set of labeled rewrite rules of the form $L \xrightarrow{\lambda} R$, where $L, R \subseteq F$, and $\lambda \in L$;
- $A$ is a set of axioms of the form $\bigwedge_{i=1}^{j} p_i \supset c$, where $p_1, \ldots, p_j, c \in F$;
- $I$ and $G$ are respectively the initial state and a boolean formula representing the bad states.
Protocol Insecurity Problems with Axioms (2)

A PIP with axioms represents a state transition system in which:

- **States**: set of facts $S$ (i.e. $S \subseteq \mathcal{F}$) such that $S \models \mathcal{A}$;

- **Transition Relation**: let $S$ be a state and $L \xrightarrow{\lambda} R$ be a rewrite rule, then $S \xrightarrow{\lambda} S'$ iff $L \subseteq S$ and $S' = (S \setminus L) \cup R$ is such that $S' \models \mathcal{A}$. 

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An attack to a PIP with axioms is a sequence of rules $\lambda_1, \ldots, \lambda_n$ such that $S_i \xrightarrow{\lambda_i} S_{i+1}$ for $i = 1, \ldots, n$ with $S_1 = \mathcal{I}$ and $S_n \models \mathcal{G}$.
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Attacks to a PIP with axioms can be compactly represented by means of partial-order attack.
Encoding PIP with axioms into SAT (1)

Given a PIP with axioms (without equivalence cycles) $\Xi$ and a positive integer $n$, we build a propositional formula $\Phi^n_\Xi$ such that any model of $\Phi^n_\Xi$ corresponds to a partial-order attack of $\Xi$. 
Encoding PIP with axioms into SAT (1)

Given a PIP with axioms (without equivalence cycles) $\Xi$ and a positive integer $n$, we build a propositional formula $\Phi_n^{\Xi}$ such that any model of $\Phi_n^{\Xi}$ corresponds to a partial-order attack of $\Xi$.

To do so, we:

1. add an additional time-index parameter to each rule $\lambda$ or fact $p$, to indicate the state at which time the rule begins or the fact holds.

2. build $\Phi_n^{\Xi}$ by unfolding $n$ times the transition relation:

   $$\Phi_n^{\Xi} = I(p^0) \land \bigwedge_{i=0}^{n-1} T_i(p^i, \lambda^i, p^{i+1}) \land G(p^n)$$

where $I$, $T$ and $G$ are formulae defining the initial state, the transition relation and the goal states, respectively.
Encoding PIP with axioms into SAT (2)

The encoding of PIP with axioms into a SAT formulae can be done in a variety of ways (see [1,2]).

The main differences between them are reflected in the formula representing the transition relation: \( \bigwedge_{k=0}^{n-1} T_i(p^i, \lambda^i, p^{i+1}) \).

By introducing axioms, significant changes must be done on the encodings.

We have adapted and extended the following two for supporting axioms:

- **Linear** encoding, and
- **Graphplan-based** encoding.

Linear Encoding with Axioms (1)

The formula $T_i(p^i, \lambda^i, p^{i+1})$ for $i = 0, \ldots, n-1$ is given by the conjunction of the following:

**Universal Formulae:** for each rewrite rule $\lambda \in \mathcal{L}$ s.t. $(L \xrightarrow{\lambda} R) \in \mathcal{R}$

\[
\lambda^i \supset \bigwedge \{ p^i \mid p \in L \} \\
\lambda^i \supset \bigwedge \{ p^{i+1} \mid p \in R \setminus L \} \\
\lambda^i \supset \bigwedge \{ \neg p^{i+1} \mid p \in L \setminus R \}
\]

**Cardinality:** $O(n|\mathcal{L}|r)$, where $r$ max #facts in a rule (usually small).

**Axioms Formulae:** for each $(p_1 \wedge \cdots \wedge p_j \supset c) \in \mathcal{A}$

\[
(p_1^i \wedge \cdots \wedge p_j^i) \supset c^i
\]

**Cardinality:** $O(n|\mathcal{A}|)$. 
Linear Encoding with Axioms (2)

Explanatory Frame Formulae with Axioms: for all facts $f \in \mathcal{F}$

$$(\neg f^i \land f^{i+1}) \supset \left( \bigvee \left\{ \lambda^i \mid (L \xrightarrow{\lambda} R) \in \mathcal{R}, f \in (R \setminus L) \right\} \lor \bigvee \left\{ p_1^{i+1} \land \cdots \land p_j^{i+1} \mid (p_1 \land \cdots \land p_j \supset f) \in \mathcal{A} \right\} \right)$$

$$(f^i \land \neg f^{i+1}) \supset \left( \bigvee \left\{ \lambda^i \mid (L \xrightarrow{\lambda} R) \in \mathcal{R}, f \in (L \setminus R) \right\} \lor \bigvee \left\{ \neg p_1^{i+1} \land p_2^{i+1} \land \cdots \land p_j^{i+1} \mid \neg p_1 \land p_2 \land \cdots \land p_j \supset \neg f \in \hat{\mathcal{A}} \right\} \right)$$

where $\hat{\mathcal{A}}$ is the set of contraposed axioms. E.g. $\neg b \supset \neg a$ is the contraposed of $a \supset b$. **Cardinality:** $O(n|\mathcal{F}| + nt|\mathcal{A}|)$, where $t$ is the max number of preconditions in an axiom (usually small).
Linear Encoding with Axioms (3)

**Conflict Exclusion Formulae with Axioms:** for all distinct rule $\lambda_1, \lambda_2$ such that $(L_1 \xrightarrow{\lambda_1} R_1) \in \mathcal{R}$, $(L_2 \xrightarrow{\lambda_2} R_2) \in \mathcal{R}$ with $L_1 \cap \text{dep}_A(L_2 \setminus R_2) \neq \emptyset$ or $L_2 \cap \text{dep}_A(L_1 \setminus R_1) \neq \emptyset$

$$\neg(\lambda_1^i \land \lambda_2^i)$$

where $\text{dep}_A(L_j \setminus R_j)$ ($j = 1, 2$) is the set of facts from which all the facts deleted by $\lambda_i$ possibly depend wrt $\mathcal{A}$. E.g. let $\mathcal{A} = \{a \supset b, b \land c \supset d\}$, then

$$\text{dep}_A(\{b\}) = \{a, b\}$$
$$\text{dep}_A(\{c\}) = \{c\}$$
$$\text{dep}_A(\{d\}) = \{a, b, c, d\}$$

**Cardinality:** $O(n|\mathcal{L}|^2)$. 
Implementation: SATMC

**SATMC v1.0:**
- input specification in IF v.1 language;
- set of **optimizing transformations** to get encodings of manageable size;
- linear encoding with iterative deepening on the number of steps.

**SATMC v2.0:**
- input specification in IF v.2 language;
- abstraction/refinement strategy based on neglecting mutex relations;
- an optimized graphplan-based encoding;
- support **axioms**.

Download it at: http://www.mrg.dist.unige.it/satmc
Implementation: Architecture

security problem (IF)

max
abs/ref
enc
solver
fail

k = 0

SAT Compiler
Linear Encoding
Graphplan–based Encoding

formula (DIMACS)

Yes
k < max
k = k + 1
unsat
No

ppop2attack
Yes
feasible?
ppop
model2ppop
model
refinement
refinement clauses

Chaff
SIM
SATO

ppop
attack
fail

Cork, 04 Jul 2004 ARSPA Workshop - IJCAR Luca Compagna
### Experimental Results on C/J

#### DY Optimized DY

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<th>Protocol</th>
<th>N</th>
<th>Atoms</th>
<th>Clauses</th>
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<th>Atoms</th>
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#### Linear Encoding

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#### Graphplan-based Encoding
Conclusions and Perspectives

- Proposed an **optimized intruder model** for SAT-based model-checking of security protocols.

- Encodings schemes extended for supporting the specification of set of axioms (without equivalence cycles).

- Up to 40% shorter attacks and up to 50% smaller SAT formulae.
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- Up to 40% shorter attacks and up to 50% smaller SAT formulae.

- Investigate and extend our approach for encoding generic set of axioms also specifying equivalence cycles: algebraic equations (e.g. exponentiation in the Diffie-Hellman protocol).

- Experiment such an optimization against industrial-scale security protocols: a considerable number of intruder knowledge manipulations can be required.
Thanks for your attention